Math 17B
Vogler
Discussion Sheet 1

1.) Determine the most general anti-derivative for each function by "guessing and checking."

a.) 5 b.) $x^2 - 3x + 5$ c.) $(x+1)(x^2-3)$ d.) $\dfrac{x^3 - x + 7}{x^2}$

e.) $\sin 3x + 3\sin x$ f.) $\sec^2 7x + \sec 3x \tan 3x$ g.) $\sec^2 x \cdot \tan^2 x$

h.) $(\sec x - \tan x)^2$ i.) $e^{3x} - x^2 e^{x^3} + e$ j.) $3^{x-2} + 5 \cdot 4^{3x}$

k.) $(1 + 2^x)(1 - 3^x)$ l.) $\dfrac{1 + 2^x}{3^x}$ m.) $x^2 \cos x + 2x \sin x$

2.) Evaluate the following sums.

a.) $\sum_{i=1}^{n} 9$ b.) $\sum_{i=1}^{1053} 9$ c.) $\sum_{i=34}^{867} 9$ d.) $\sum_{i=1}^{50} i(2i+3)$ e.) $\sum_{i=1}^{60} (5i - i^2)$

f.) $\sum_{i=26}^{62} (5i - i^2)$ g.) $\sum_{i=1}^{30} (\ \ln(i+2) - \ln(i+1)\ )$ h.) $\sum_{i=1}^{4} \cos \pi i$ i.) $\sum_{i=1}^{17} \cos \pi i$

j.) $\sum_{i=1}^{n} \cos \pi i$

3.) Prove that $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$.

4.) Evaluate the following sums.

a.) $1 + 2 + 3 + 4 + \cdots + 75$ b.) $151 + 152 + 153 + \cdots + 364$

c.) $1 + 2 + 4 + 8 + 16 + \cdots + 1,048,576$ d.) $1.1^6 + 1.1^7 + 1.1^8 + \cdots + 1.1^{200}$

5.) a.) Sketch the graph of $y = 3x^2 + 2$ on the interval [0, 1].

b.) Estimate the area of the region below the graph of $y = 3x^2 + 2$ and above [0, 1] using rectangles above three equal subdivisions and

i.) left endpoints of the subdivisions.

ii.) right endpoints of the subdivisions.

iii.) midpoints of the subdivisions.

6.) Consider the following sequence of numbers : $1, 5, 9, 13, 17, 21, 25, \cdots$. Find the sum of the 61st through the 127th numbers in this sequence.

7.) Consider the following sequence of numbers : $1, 2, 5, 10, 17, 26, 37, 50, 65, \cdots$. Find the sum of the first 200 numbers in this sequence.

8.) A Sooper Dooper Ball is thrown straight up from ground level and reaches a height of 100 feet before it falls back to earth and begins bouncing. On each bounce it reaches a height equal to 95% of the distance it fell.

a.) How high is the 6th bounce ? the 35th bounce ?

b.) What is the total distance that the ball travels through the air as it strikes the ground for the 50th time ?

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

9.) Use the fact that $1 + r + r^2 + r^3 + \cdots + r^n = \dfrac{1 - r^{n+1}}{1 - r}$ to derive a formula for $1 + 2 \cdot r + 3 \cdot r^2 + 4 \cdot r^3 + \cdots + (n+1) \cdot r^n$. What is the value of $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 31 \cdot 2^{30}$?

Thanks to Dr. Kouba