Math 17B
Vogler
Discussion Sheet 5

1.) Solve the following differential equations.

a.) $\dfrac{dy}{dx} = xe^x$ b.) $\dfrac{dy}{dx} = xy\sqrt{y-4}$ c.) $\dfrac{dy}{dx} = \sin^3 x \cos^2 y$

d.) $\dfrac{dy}{dx} = y^2(y-1)$ e.) $\dfrac{dy}{dx} = \sin x \cos y$ f.) $\dfrac{dy}{dx} = xy - y + 3x - 3$

g.) $\dfrac{dy}{dx} = \dfrac{x + xy^3 + 1 + y^3}{xy^2 - 2y^2}$ h.) $\dfrac{dy}{dx} = e^{2x+3y}$ and $y(1) = 0$

2.) Let M be the total mass (in grams) of a black bullhead (a sport fish common throughout Minnesota's lakes with sandy, muddy bottoms) at time t (in years). Assume that its growth rate is given by $\dfrac{dM}{dt} = (1/100)M(400 - M)$. If $M(0) = 2$ grams, solve the D.E. and solve explicitly for mass M. What is the bullhead's mass in 1 year ? in 2 years ? Determine an upper limit (asymptotic mass) for the mass of this fish.

3.) Solve Problem B on page 2 of this discussion sheet.

4.) Consider the function $f(x) = \cos 2x$ on the interval $[0, 1/2]$. What should n be in order that the Taylor Polynomial of degree n centered at $x = 0$ have a Taylor Error of at most 0.0001 ?

5.) Consider the function $f(x) = \dfrac{x}{x+1}$ on the interval $[0, 3/4]$. What should n be in order that the Taylor Polynomial of degree n centered at $x = 0$ have a Taylor Error of at most 0.0001 ?

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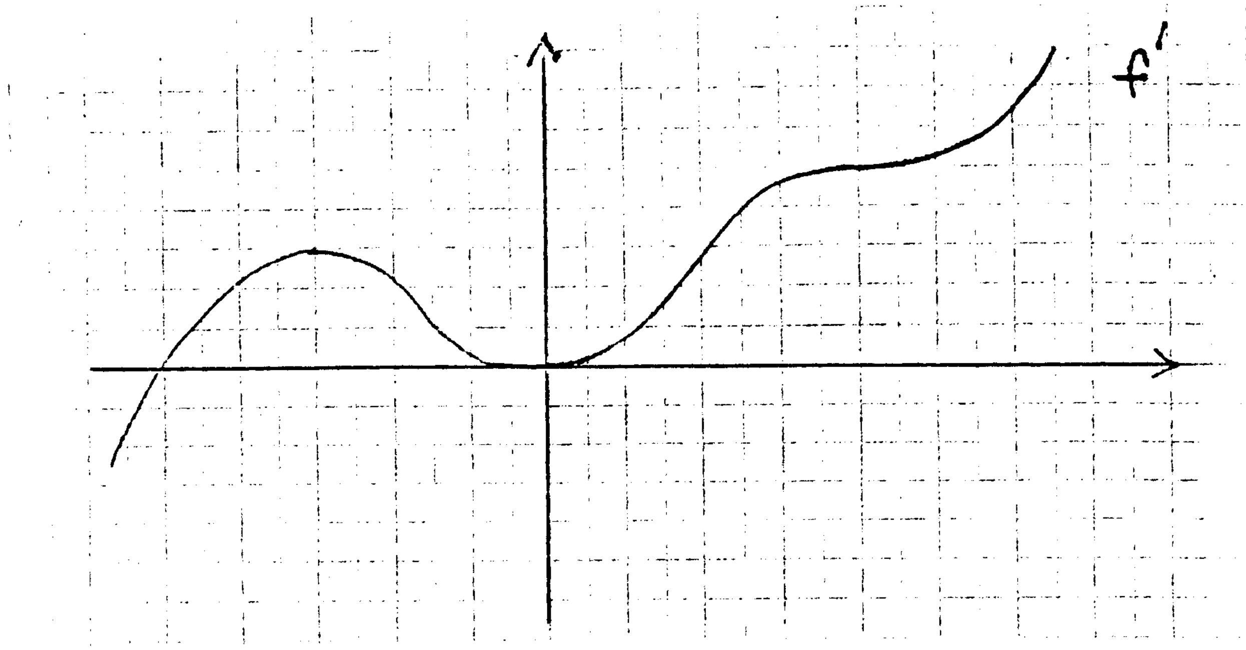
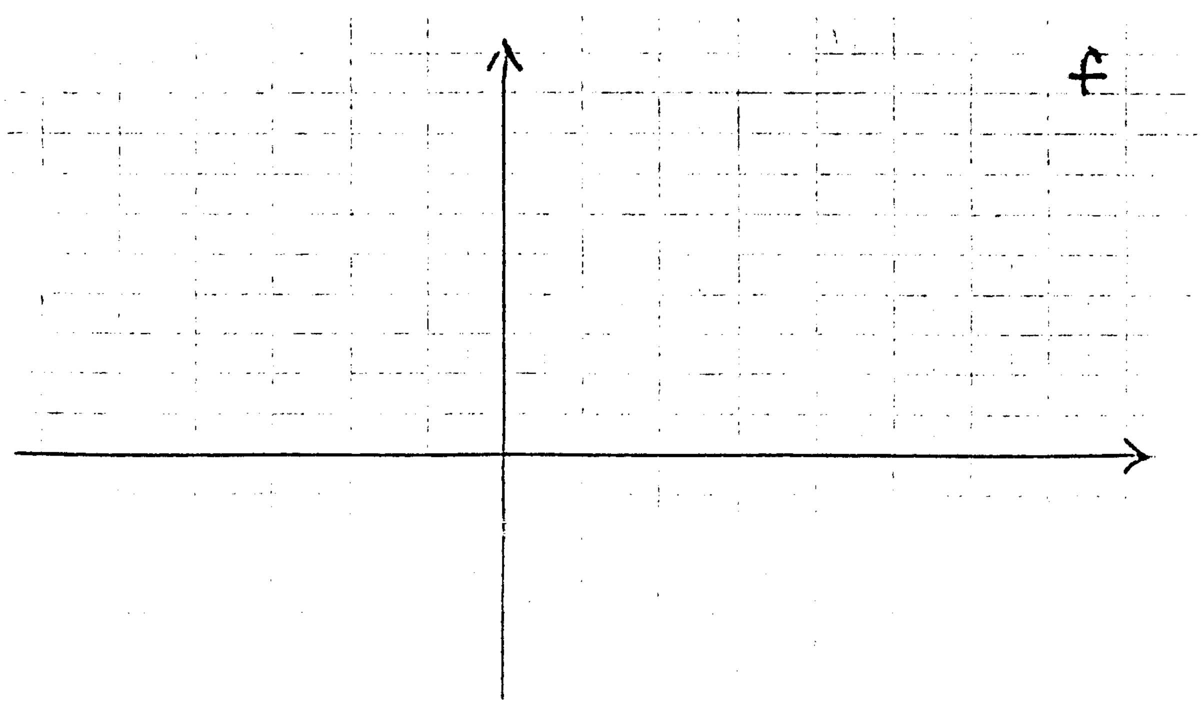
THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

6.) You have 8 black socks, 12 blue socks, 10 gray socks, and 5 white socks randomly scattered in your bureau drawer. If you reach into the drawer without looking, how many socks must you take out to be sure of having a matching pair of socks ? a matching pair of white socks ?

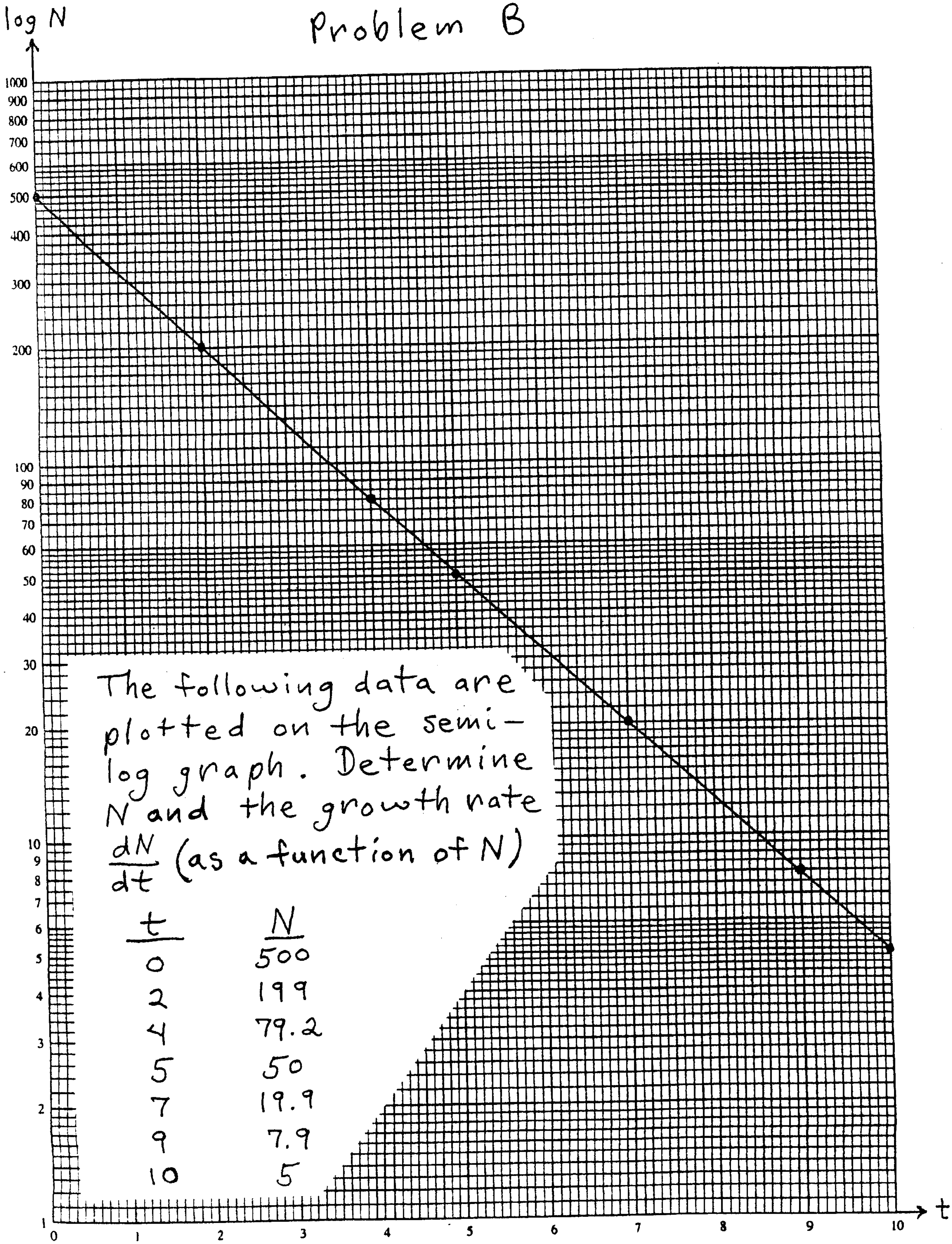
1.) The graph of a derivative f' is given below. Set up a sign chart for the second derivative f'' and sketch a graph of the function f, indicating extrema and inflection points.





Thanks to Dr. Kouba

Problem B



The following data are plotted on the semi-log graph. Determine N and the growth rate $\frac{dN}{dt}$ (as a function of N)

t	N
0	500
2	199
4	79.2
5	50
7	19.9
9	7.9
10	5

Problem B (Solution)

assume $\log N = \log C + rt$ (a line); then $C = 500$ so $\underline{\log N = \log 500 + rt}$, and $t = 10$, $N = 5$ $\longrightarrow$

$$\log 5 = \log 500 + 10r \longrightarrow$$
$$\log 5 - \log 500 = 10r \longrightarrow$$
$$\log \frac{5}{500} = 10r \longrightarrow \log \frac{1}{100} = 10r \longrightarrow$$
$$\log 10^{-2} = 10r \longrightarrow -2 = 10r \longrightarrow$$
$$r = -1/5$$; then

$$\log N = \log 500 - \frac{1}{5}t \longrightarrow$$
$$10^{\log N} = 10^{\log 500 - \frac{1}{5}t} \longrightarrow$$
$$N = 10^{\log 500}\, 10^{-\frac{1}{5}t} \longrightarrow$$
$$\boxed{N = 500 \cdot 10^{-\frac{1}{5}t}}$$; then

$$\frac{dN}{dt} = \underbrace{500 \cdot 10^{-\frac{1}{5}t}}_{N} \cdot \ln 10 \cdot \frac{-1}{5} \longrightarrow$$

$$\boxed{\frac{dN}{dt} = \frac{-\ln 10}{5} N}$$