Math 17B
Vogler
Discussion Sheet 9

1.) Consider the matrix $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. For each of the following vectors X, compute RX and plot X and RX on the same x_1x_2-coordinate sytem.

a.) $X = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ b.) $X = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ c.) $X = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ d.) $X = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

e.) Make a conjecture about how vector RX is related to vector X .

2.) Consider the matrix $R = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$. Is this matrix a rotation matrix ? If so, in what direction and how many radians does it rotate vectors ?

3.) Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be the $2x2$ identity matrix and let λ be a constant (real or imaginary). Solve the matrix equation $det(A - \lambda I) = 0$ for λ for each of the following matrices A.

a.) $A = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$ b.) $A = \begin{pmatrix} 14 & 16 \\ -9 & -10 \end{pmatrix}$ c.) $A = \begin{pmatrix} 3 & 2 \\ -5 & 1 \end{pmatrix}$ d.) $A = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

4.) Find eigenvalues and the corresponding eigenvectors for each matrix.

a.) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ b.) $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ c.) $\begin{pmatrix} 13 & -4 \\ -4 & 7 \end{pmatrix}$

5.) Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ from problem 4.)a.) with eigenvectors V_1 and V_2 corresponding to distinct eigenvalues.

a.) Write the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ as a linear combination of V_1 and V_2 , i.e., determine constants c_1 and c_2 so that $c_1V_1 + c_2V_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

b.) Use your results in part a.) to compute $A^{50}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

6.) Consider the Leslie matrix $L = \begin{pmatrix} 3/2 & 3 \\ 5/6 & 0 \end{pmatrix}$. Find eigenvalues λ_1 and λ_2 with eigenvectors V_1 and V_2 corresponding to distinct eigenvalues. Note that one eigenvalue, say λ_1, is positive and the other, λ_2, is negative, with $|\lambda_2| < \lambda_1$.

a.) Assume that $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ is the initial age distribution. Now write the vector $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ as a linear combination of V_1 and V_2 , i.e., determine constants c_1 and c_2 so that $c_1V_1 + c_2V_2 = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

b.) Use your results in part a.) to determine the age distribution at the end of season $t = 10$, i.e., compute $L^{10} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

7.) Following are Leslie matrices. Find both eigenvalues, determine if the population is increasing or decreasing, and find the stable age distribution for each matrix.

a.) $\begin{pmatrix} 1 & 3 \\ 0.7 & 0 \end{pmatrix}$ b.) $\begin{pmatrix} 0 & 1 \\ 0.81 & 0 \end{pmatrix}$

" Wisdom outweighs any wealth."– Sophocles

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