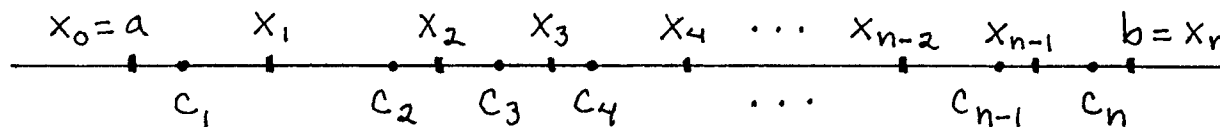


Math 17B
 Vogler
 Limit Definition of the Definite Integral

- 1.) Consider a function $y = f(x)$ defined on the closed interval $[a, b]$.
- 2.) *Partition* the interval $[a, b]$ into n pieces of any size :

$$x_0 = a, x_1, x_2, x_3, \dots, x_{n-1}, b = x_n$$

Define the *mesh* of the partition to be : $\max_{1 \leq i \leq n} (x_i - x_{i-1})$. (In other words, the mesh of a partition is the length of the largest subinterval.)



- 3.) Pick *sampling points* $c_1, c_2, c_3, \dots, c_{n-1}, c_n$, where c_i is a number in the subinterval $[x_{i-1}, x_i]$ for $i = 1, 2, 3, \dots, n$. Let $\Delta x_i = x_i - x_{i-1}$ be the length of the subinterval $[x_{i-1}, x_i]$ for $i = 1, 2, 3, \dots, n$.

- 4.) Compute the *Riemann Sum* defined by $\sum_{i=1}^n f(c_i) \cdot \Delta x_i$

- 5.) The *Definite Integral* is then given to be

$$\int_a^b f(x) dx = \lim_{mesh \rightarrow 0} \sum_{i=1}^n f(c_i) \cdot \Delta x_i$$

REMARK : The above is a formal and general definition of the Definite Integral. When doing problems using this step-by-step process for computing $\int_a^b f(x) dx$, it is convenient to

- i.) pick equally-spaced partition points so that all of the subintervals have the same length : $\Delta x_i = \frac{b-a}{n}$ for $i = 1, 2, 3, \dots, n$;

- ii.) pick sampling points to be the right-hand endpoints of the subintervals : $c_i = a + \frac{b-a}{n} \cdot i$ for $i = 1, 2, 3, \dots, n$; then

- iii.) $\int_a^b f(x) dx = \lim_{mesh \rightarrow 0} \sum_{i=1}^n f(c_i) \cdot \Delta x_i = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \cdot \Delta x_i$.