Math 17B
Vogler
Leslie Matrices - for Population Models with Discrete Breeding Seasons

We now discuss populations with discrete breeding seasons, where reproduction is limited to a particluar season of the year. For example, let's consider the number of female ruby-throated hummingbirds in a population with an annual breeding season of March to July. A female usually lays one clutch of two eggs; sometimes two clutches are laid. We will assume that the average life span of a female hummingbird is four years. We define the age of a bird at the END of a breeding season as follows :

age zero (0) : any bird that is born during the current breeding season
age one (1) : any zero-year old bird which survives to the end of the next breeding season
age two (2) : any one-year old bird which survives to the end of the next breeding season
age three (3) : any two-year old bird which survives to the end of the next breeding season

Let $N_x(t)$ represent the total number of female hummingbirds of age x at the end of breeding season t for $t = 0, 1, 2, 3, 4, 5, \cdots$. We make the following assumptions about reproductive viability of female birds:

age zero (0) : not yet reproductively mature
age one (1) : will produce an average of 1.2 female offspring the next breeding season which survive
age two (2) : will produce an average of 1.5 female offspring the next breeding season which survive
age three (3) : will produce an average of 0.7 female offspring the next breeding season which survive

This can be summarized in the following equation :

$$N_0(t+1) = (1.2)N_1(t) + (1.5)N_2(t) + (0.7)N_3(t)$$

We make the following assumptions about the survival rates of female birds:

50% of age zero (0) females at time t survive to time $t+1$;

35% of age one (1) females at time t survive to time $t+1$;
15% of age two (2) females at time t survive to time $t+1$;
0% of age three (3) females at time t survive to time $t+1$.

This can be summarized in the following equations:

$$N_1(t+1) = (0.5)N_0(t) \ ,$$
$$N_2(t+1) = (0.35)N_1(t) \ ,$$
and
$$N_3(t+1) = (0.15)N_2(t) \ .$$

Let $N(t)$ represent the total number of female hummingbirds at the end of breeding season t for $t = 0, 1, 2, 3, 4, 5, \cdots$. Using matrix notation we get the following representations :

Let $N(t) = \begin{pmatrix} N_0(t) \\ N_1(t) \\ N_2(t) \\ N_3(t) \end{pmatrix}$ represent the total number of female hummingbirds by age group at the end of season t .

Let $N(t+1) = \begin{pmatrix} N_0(t+1) \\ N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \end{pmatrix}$ represent the total number of female hummingbirds by age group at the end of season $t+1$.

Let $L = \begin{pmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{pmatrix}$. Combining these matrices gives

$$\begin{pmatrix} N_0(t+1) \\ N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \end{pmatrix} = \begin{pmatrix} 0 & 1.2 & 1.5 & 0.7 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.35 & 0 & 0 \\ 0 & 0 & 0.15 & 0 \end{pmatrix} \begin{pmatrix} N_0(t) \\ N_1(t) \\ N_2(t) \\ N_3(t) \end{pmatrix} \text{, i.e.,}$$

$$N(t+1) = L \ N(t) \ .$$

The 4 by 4 matrix L is called a Leslie Matrix.