Math 17B
Vogler

Arc length Example

<u>Fact</u> One can show that the arc length of a continuous function $y=f(x)$ from $x=a$ to $x=b$ is

$$\boxed{ARC = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$$

<u>Ex</u> Compute the length of the graph of $y=1+\frac{2}{3}x^{3/2}$ on the interval $[0,3]$.

$$y = 1+\frac{2}{3}x^{3/2} \underset{D}{\Longrightarrow} y' = x^{\frac{1}{2}}$$

$$\Rightarrow ARC = \int_0^3 \sqrt{1+(y')^2}\,dx = \int_0^3 \sqrt{1+(\sqrt{x})^2}\,dx = \int_0^3 \sqrt{1+x}\,dx$$

$$= \int_0^3 (1+x)^{1/2}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^3 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$

$$= \frac{2}{3}\cdot 8 - \frac{2}{3} = \boxed{\frac{14}{3}}.$$