

Differential Equations

Defn A first-order differential equation is an equation containing a first derivative.

Types of First-Order Differential Equations (D.E.)

I) Pure Time D.E. (x only on R.H.S.)

Ex $\frac{dy}{dx} = x^2 + e^x$

II) Autonomous D.E. (y only on R.H.S.)

Ex $\frac{dy}{dx} = y^2 - y$

III) Seperable D.E. (both x & y on R.H.S., but the functions of x, y are connected through multiplication or division)

Ex $\frac{dy}{dx} = x^3 \sin y$

IV) First-Order Linear (F.O.L.) D.E. is of the

form $\frac{dy}{dx} + \underbrace{p(x)}_{\text{Fns of x only!}} y = \underbrace{q(x)}_{\text{Fns of x only!}}$

Ex $\frac{dy}{dx} - xy = x$

Specific D.E. Applications

I) If N is population size at time t , then the D.E.

$$\frac{dN}{dt} = 3N$$

represents the growth rate of the population.

When the growth rate is of the form

$$\frac{dN}{dt} = N\left(1 - \frac{N}{20}\right)$$

it is called a Logistic Growth Equation (per capita growth rate is density dependent; see pg. 486)

II) If L is the distance traveled at time t ,

then

$$\frac{dL}{dt} = t^2 + t$$

represents the instantaneous velocity of the object at time t .