Math 17B
Vogler

Integration by Parts

Recall: Product Rule is

$$D[f(x)\cdot g(x)] = f'(x)g(x) + f(x)g'(x).$$

Then, we have

$$\int D[f(x)\cdot g(x)]\,dx = \int g(x)f'(x)\,dx + \int f(x)g'(x)\,dx$$

$$\left(\begin{array}{l} \text{Let } u = f(x) \overset{D}{\Rightarrow} du = f'(x)\,dx, \text{ and} \\ \text{let } v = g(x) \overset{D}{\Rightarrow} dv = g'(x)\,dx \end{array}\right)$$

$$\Rightarrow f(x)g(x) = \int v\,dv + \int u\,dv \Rightarrow uv = \int v\,du + \int u\,dv$$

$$\Rightarrow \boxed{\int u\,dv = uv - \int v\,du}$$ Integration by Parts Formula

Moral: Integration by parts is the integral's 'response' to the chain rule of differentiation.

Ex $\int xe^{2x}\,dx$

Let $u = x$, $dv = e^{2x}dx$

$\Rightarrow du = 1\cdot dx$, $v = \frac{1}{2}e^{2x}$

$$= x\left(\tfrac{1}{2}e^{2x}\right) - \int \tfrac{1}{2}e^{2x}\,dx$$

$$= \tfrac{1}{2}xe^{2x} - \tfrac{1}{2}\cdot\tfrac{1}{2}e^{2x} + C$$