Math 17B
Vogler

Rules for Matrix Row Reduction

i) Switch rows.
ii) Multiply row by non zero constant.
iii) Add a multiple of one row to another row.

Goal in using these rules: Create 1's along the diagonal and 0's above and below these 1's, and **DO NOT** undo the 0's already created.

Solving System of Equations

Ex Solve for x, y, z using a augmented matrix

$$2x + 2y + 2z = 4$$
$$2x - y - z = 1$$
$$5x + y - 2z = -3$$

1) Create the augmented matrix

$$\Rightarrow \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 2 & 2 & 2 & 4 \\ 2 & -1 & -1 & 1 \\ 5 & 1 & -2 & -3 \end{array}\right]$$

2) Use Rule ii) to make 1st pivot a 1 (i.e $R_1 = \frac{1}{2}R_1$)

$$\underset{(2)}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & -1 & -1 & 1 \\ 5 & 1 & -2 & -3 \end{array}\right] \underset{(3)}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -3 & -3 & -3 \\ 0 & -4 & -7 & -13 \end{array}\right]$$

3) Use Rule iii) to make all numbers below the 1st pivot 0's (i.e. $R_2 = R_2 + (-2)R_1$ & $R_3 = R_3 + (-5)R_3$)

4) Use Rule ii) to make 2nd pivot a 1 (i.e. $R_2 = -\frac{1}{3}R_2$)

$$\underset{(4)}{\sim}\begin{bmatrix} 1 & 1 & 1 & \vdots & 2 \\ 0 & 1 & 1 & \vdots & 1 \\ 0 & -4 & -7 & \vdots & -13 \end{bmatrix} \underset{(5)}{\sim} \begin{bmatrix} 1 & 1 & 1 & \vdots & 2 \\ 0 & 1 & 1 & \vdots & 1 \\ 0 & 0 & -3 & \vdots & -9 \end{bmatrix} \underset{(6)}{\sim} \begin{bmatrix} 1 & 1 & 1 & \vdots & 2 \\ 0 & 1 & 1 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

5) Use Rule iii) to make all numbers below the 2nd pivot 0's (i.e. $R_3 = R_3 + 4R_2$)

6) Use Rule ii) to make 3rd pivot a 1 (i.e. $R_3 = -\frac{1}{3}R_3$)

7) Use Rule iii) to make all numbers above the 2nd pivot 0's (i.e. $R_1 = R_1 + (-1)R_2$)

$$\underset{(7)}{\sim}\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 1 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} \underset{(8)}{\sim} \begin{array}{c} \begin{array}{cccc} x & y & z & \# \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -2 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} \end{array} \Rightarrow \boxed{\begin{array}{l} x = 1 \\ y = -2 \\ z = 3 \end{array}}$$

8) Use Rule iii) to make all numbers above the 3rd pivot 0's (i.e. $R_2 = R_2 + (-1)R_3$)

9) Convert the augmented matrix back into a system of equations, and you are finished.

Notes: a) There can be variations with the order of steps 1)−9) which might allow one to get to the answer more effectively.

b) For more complicated problems, your 2nd pivot will be a zero. For this, you need to use Rule i) to obtain a non zero pivot, usually from the 3rd row.

c) For even more complicated problems, you will get a nonzero number equals zero, which means there's no solution, or a row of all zeros, which means there's infinitely many solutions.