

Types of Partial Fractions

Problems with Examples

I) Distinct Linear Factors

$$\text{Ex 1) } \int \frac{x-3}{x^2+2x-3} dx = \int \frac{x-3}{(x-1)(x+3)} dx$$

$$= \int \frac{A}{x-1} + \frac{B}{x+3} dx$$

$$2) \int \frac{x^2+x+1}{x^3-4x} dx = \int \frac{x^2+x+1}{x(x-2)(x+2)} dx$$

$$= \int \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} dx$$

II) Repeated Linear Factors

$$\text{Ex 1) } \int \frac{4-3x}{x^2(x+2)} dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} dx$$

$$2) \int \frac{x^3+7}{x(x-1)^3} dx = \int \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} dx$$

III) Irreducible Quadratic

$$\text{Ex 1) } \int \frac{3x-5}{x(x^2+1)} dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+1} dx$$

$$2) \int \frac{x^2+x+1}{(x^2+4)x} dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+4} dx$$

IV) Polynomial Division First

Note: Only necessary if degree of the numerator is equal to or larger than the degree of the denominator.

$$\begin{aligned} \text{Ex 1) } & \int \frac{x^3 + x^2 - 3}{x^2 - 1} dx \\ &= \int x + 1 + \frac{x - 2}{x^2 - 1} dx \\ &= \frac{1}{2}x^2 + x + \int \frac{x - 2}{(x+1)(x-1)} dx \\ &= \frac{1}{2}x^2 + x + \int \frac{A}{x+1} + \frac{B}{x-1} dx \end{aligned}$$

$$\begin{array}{r} x+1 \\ x^2-1 \overline{) x^3+x^2-3} \\ \underline{-(x^3-x)} \\ x^2+x-3 \\ \underline{-(x^2-1)} \\ x-2 \end{array}$$

$$\begin{aligned} 2) & \int \frac{x^2 + 4}{x^2 + x} dx \\ &= \int 1 + \frac{-x + 4}{x^2 + x} dx \\ &= x + \int \frac{4 - x}{x(x+1)} dx \\ &= x + \int \frac{A}{x} + \frac{B}{x+1} dx \end{aligned}$$

$$\begin{array}{r} 1 \\ x^2+x \overline{) x^2+4} \\ \underline{-(x^2+x)} \\ -x+4 \end{array}$$