Section 5.8

1.) $\int (4x^2 - x)\,dx = 4 \cdot \frac{1}{3}x^3 - \frac{1}{2}x^2 + c$

8.) $\int (x - 2x^2 - 3x^3 - 4x^4)\,dx$

$= \frac{1}{2}x^2 - 2\cdot\frac{1}{3}x^3 - 3\cdot\frac{1}{4}x^4 - 4\cdot\frac{1}{5}x^5 + c$

10.) $\int \left(x^2 - \frac{2}{x^2} + \frac{3}{x^3}\right)dx = \int (x^2 - 2x^{-2} + 3x^{-3})\,dx$

$= \frac{1}{3}x^3 - 2\cdot\frac{1}{-1}x^{-1} + 3\cdot\frac{1}{-2}x^{-2} + c$

17.) $\int \frac{1}{1+2x}\,dx = \frac{1}{2}\cdot \ln|1+2x| + c$

19.) $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + c$

20.) $\int \left(e^{\frac{1}{2}x} + e^{\frac{-1}{2}x}\right)dx = \frac{1}{1/2}e^{1/2 x} + \frac{1}{-1/2}e^{-\frac{1}{2}x} + c$

22.) $\int -3e^{-4x}\,dx = -3\cdot\frac{1}{-4}e^{-4x} + c$

24.) $\int \frac{3}{e^{-x}}\,dx = \int 3e^{x}\,dx = 3e^{x} + c$

25.) $\int \sin 2x\,dx = \frac{-1}{2}\cos 2x + c$

28.) $\int \left[\cos\left(\frac{1}{5}x\right) - \sin\left(\frac{1}{5}x\right)\right]dx$

$= \frac{1}{1/5}\sin\left(\frac{1}{5}x\right) - \frac{-1}{1/5}\cos\left(\frac{1}{5}x\right) + c$

31.) $\int \sec^2(2x)\,dx = \frac{1}{2}\tan(2x) + c$

33.) $\int \sec^2\left(\frac{1}{3}x\right) dx = \frac{1}{1/3} \tan\left(\frac{1}{3}x\right) + c$

35.) $\int \frac{\sec x + \cos x}{\cos x}\, dx = \int \left(\frac{\sec x}{\cos x} + \frac{\cos x}{\cos x}\right) dx$

$= \int \left(\frac{1}{\cos^2 x} + 1\right) dx = \int (\sec^2 x + 1)\, dx$

$= \tan x + x + C$

39.) $\int \left[\sec^2(3x-1) + \frac{x^2-3}{x}\right] dx$

$= \int \left[\sec^2(3x-1) + x - 3\cdot\left(\frac{1}{x}\right)\right] dx$

$= \frac{1}{3}\tan(3x-1) + \frac{1}{2}x^2 - 3\cdot \ln|x| + c$

41.) $\int \frac{e^{(a+1)x}}{a}\, dx = \frac{1}{a}\cdot\frac{1}{a+1}\cdot e^{(a+1)x} + c$

42.) $\cos 2\theta = 1 - 2\sin^2\theta \rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$;

$\int \sin^2(a^2x+1)\, dx = \int \frac{1}{2}\left(1 - \cos(2(a^2x+1))\right) dx$

$= \frac{1}{2}\int \left(1 - \cos(2a^2x+2)\right) dx$

$= \frac{1}{2}\left(x - \frac{1}{2a^2}\sin(2a^2x+2)\right) + c$

43.) $\int \frac{1}{ax+3}\, dx = \frac{1}{a}\ln|ax+3| + c$

44.) $\int \frac{a}{a+x}\, dx = \int a\cdot\frac{1}{a+x}\, dx$

$= a\cdot\ln|a+x| + c$

45.) $\int (x^{a+2} - a^{x+2})\, dx$

$= \frac{1}{a+3} x^{a+3} - \frac{1}{\ln a} a^{x+2} + C$

46.) $\int \frac{e^{-ax} + e^{ax}}{2a}\, dx = \frac{1}{2a}\left(\frac{1}{-a} e^{-ax} + \frac{1}{a} e^{ax}\right) + C$

49.) $\frac{dy}{dx} = x(1+x) = x + x^2 \xrightarrow{\text{anti.}}$

$y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

50.) $\frac{dy}{dx} = e^{-4x} \xrightarrow{\text{anti.}} y = \frac{1}{-4} e^{-4x} + C$

54.) $\frac{dy}{dt} = 1 - e^{-3t} \xrightarrow{\text{anti.}} y = t - \frac{1}{-3} e^{-3t} + C$

60.) $\frac{dy}{dx} = \frac{1}{3}x^2 \xrightarrow{\text{anti.}} y = \frac{1}{3} \cdot \frac{1}{3} x^3 + C = \frac{1}{9}x^3 + C$

and $x = 0$, $y = 2 \longrightarrow 2 = \frac{1}{9}(0)^3 + C \longrightarrow C = 2$

$\longrightarrow y = \frac{1}{9}x^3 + 2$

66.) $\frac{dW}{dt} = e^{-3t} \xrightarrow{\text{anti.}} W = \frac{1}{-3} e^{-3t} + C$ and

$t = 0$, $W = 2 \longrightarrow 2 = \frac{-1}{3} \cancel{e^0}^{1} + C \longrightarrow$

$C = 2 + \frac{1}{3} = \frac{6}{3} + \frac{1}{3} = 7/3$ so

$W = \frac{-1}{3} e^{-3t} + 7/3$

70.) $\frac{dT}{dt} = \cos(\pi t) \xrightarrow{\text{anti.}} T = \frac{1}{\pi} \sin(\pi t) + C$

and $t = 0$, $T = 3 \longrightarrow 3 = \frac{1}{\pi} \cancel{\sin 0}^{0} + C$

$\longrightarrow C = 3 \longrightarrow T = \frac{1}{\pi} \sin(\pi t) + 3$

72.) $\frac{dN}{dt} = t^{-1/3} \xrightarrow{\text{anti.}} N = \frac{3}{2}t^{2/3} + C$ and

$t=0,\ N=60 \rightarrow 60 = \frac{3}{2}(0)^{2/3} + C$

$\rightarrow C = 60$ so $N = \frac{3}{2}t^{2/3} + 60$.

73.) $\frac{dL}{dx} = e^{-0.1x} \xrightarrow{\text{anti.}} L = \frac{1}{-0.1}e^{-0.1x} + C \rightarrow$

$L = -10e^{-\frac{1}{10}x} + C$ and

$\lim_{x\to\infty} \left(-10e^{-\frac{1}{10}x} + C\right) = -10 \underbrace{\cancel{e^{-\infty}}}_{0} + C = 25$

$\rightarrow L = -10e^{-\frac{1}{10}x} + 25$; if $x=0$,

then $L = -10 \cdot \underbrace{\cancel{e^0}}_{1} + 25 = 15$.

75.) $s''(t) = -32$ ft./s.^{2} $\xrightarrow{\text{anti.}}$

vel.: $s'(t) = -32t + C$ (dropped :

$t=0,\ s'(0) = 0$ ft./s. $\rightarrow$

$0 = -32(0) + C \rightarrow C = 0$) $\rightarrow$

$\boxed{s'(t) = -32t} \xrightarrow{\text{anti.}}$

height: $s(t) = -32 \cdot \frac{1}{2}t^2 + C = -16t^2 + C$

($t=0,\ s=100$ ft. $\rightarrow 100 = -16(0)^2 + C$

$\rightarrow C = 100$) $\rightarrow$ $\boxed{s(t) = -16t^2 + 100}$;

a.) hit ground : $s(t) = 0 \rightarrow -16t^2 + 100 = 0$

$\rightarrow 16t^2 = 100 \rightarrow t^2 = \frac{100}{16} \rightarrow$

$t = \frac{10}{4} = 5/2$ seconds $= 2.5$ seconds.

b.) velocity when $t = 2.5$ s. is

$s'(2.5) = -32(2.5) = -80$ ft./s. so

speed $= 80$ ft./s.

76.) $\frac{dN}{dt} = 3\sin(2\pi t) \xrightarrow{\text{anti.}}$

$N = 3 \cdot \frac{-1}{2\pi}\cos(2\pi t) + C$ and $t=0$, $N=10$

$\rightarrow 10 = \frac{-3}{2\pi}\overset{1}{\cancel{\cos 0}} + C \rightarrow C = 10 + \frac{3}{2\pi} \rightarrow$

$N = 10 + \frac{3}{2\pi} - \frac{3}{2\pi}\cos(2\pi t)$;

since $-1 \le \cos(2\pi t) \le +1$
the population fluctuates between
$N = 10 + \frac{3}{2\pi} - \frac{3}{2\pi} = 10$ (1000's) and

$N = 10 + \frac{3}{2\pi} + \frac{3}{2\pi} = 10 + \frac{3}{\pi}$ (1000's) .

77.) $V(t)$: grams of H_2O at time t (hrs.),
rate at which $V(t)$ changes is

a.) $\frac{dV}{dt} = (\text{rate in}) - (\text{rate out})$

$= \frac{4 \text{ gm.}}{\text{hr.}} - a t(24-t) \frac{\text{gm.}}{\text{hr.}}$

proportionality constant

$\rightarrow \frac{dV}{dt} = 4 - at(24-t)$;

b.) $\frac{dV}{dt} = 4 - 24at + at^2 \xrightarrow{\text{anti.}}$

$V = 4t - 12at^2 + a \cdot \frac{1}{3}t^3 + C$;

if we assume that $V(0) = V(24)$,

then

$$\cancel{c} = 4(24) - 12a(24)^2 + \frac{a}{3}(24)^3 + \cancel{c} \longrightarrow$$

$$0 = 96 - 6912a + 4608a \longrightarrow$$

$$0 = 96 - 2304a \longrightarrow$$

$$a = \frac{96}{2304} = \frac{1}{24}.$$