Section 9.4

2.) a.) $X-Y=\begin{bmatrix}-4\\3\\1\end{bmatrix}-\begin{bmatrix}0\\-2\\3\end{bmatrix}=\begin{bmatrix}-4\\5\\-2\end{bmatrix}$

b.) $2X+3Y=2\begin{bmatrix}-4\\3\\1\end{bmatrix}+3\begin{bmatrix}0\\-2\\3\end{bmatrix}$

$=\begin{bmatrix}-8\\6\\2\end{bmatrix}+\begin{bmatrix}0\\-6\\9\end{bmatrix}=\begin{bmatrix}-8\\0\\11\end{bmatrix}$

c.) $-X-2Y=-\begin{bmatrix}-4\\3\\1\end{bmatrix}-2\begin{bmatrix}0\\-2\\3\end{bmatrix}$

$=\begin{bmatrix}4\\-3\\-1\end{bmatrix}+\begin{bmatrix}0\\4\\-6\end{bmatrix}=\begin{bmatrix}4\\1\\-7\end{bmatrix}$

4.) $\overrightarrow{AB}=\begin{bmatrix}2-(-1)\\-4-0\end{bmatrix}=\begin{bmatrix}3\\-4\end{bmatrix}$

5.) $\overrightarrow{AB}=\begin{bmatrix}-1-0\\-1-1\\2-(-3)\end{bmatrix}=\begin{bmatrix}-1\\-2\\5\end{bmatrix}$

7.) $X=\begin{bmatrix}1\\3\end{bmatrix}\rightarrow |X|=\sqrt{1^2+3^2}=\sqrt{10}$

10.) $X=\begin{bmatrix}-2\\1\\-3\end{bmatrix}\rightarrow |X|=\sqrt{(-2)^2+(1)^2+(-3)^2}=\sqrt{14}$

12.) $X=\begin{bmatrix}2\\0\\-4\end{bmatrix}\rightarrow |X|=\sqrt{2^2+0^2+(-4)^2}=$

$=\sqrt{20}=\sqrt{(4)(5)}=2\sqrt{5}\rightarrow$

unit vector $U = \frac{1}{|X|} X = \frac{1}{2\sqrt{5}} \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} \\ 0 \\ -2/\sqrt{5} \end{bmatrix}$

16.) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -4 \end{bmatrix} = (-1)(-3) + (2)(-4) = -5$

17.) $\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = (0)(-3) + (-1)(1) + (3)(1) = 2$

23.) $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $Y = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \rightarrow |X| = \sqrt{1^2 + 2^2} = \sqrt{5}$,

$|Y| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$, then

$$\cos\theta = \frac{X \cdot Y}{|X||Y|} = \frac{3-2}{\sqrt{5}\sqrt{10}} = \frac{1}{\sqrt{50}} \rightarrow$$

$\theta = \arccos \frac{1}{\sqrt{50}} \approx 1.429$ radians

or $81.87°$

26.) $X = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $Y = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} \rightarrow$

$|X| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14}$, $|Y| = \sqrt{3^2 + 1^2 + (-4)^2} = \sqrt{26}$,

then $\cos\theta = \frac{X \cdot Y}{|X||Y|} = \frac{3 - 3 - 8}{\sqrt{14}\sqrt{26}} = \frac{-8}{\sqrt{364}} \rightarrow$

$\theta = \arccos\left(\frac{-8}{\sqrt{364}}\right) \approx 2.003$ radians

or $114.8°$

27.) $X = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $Y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then $X \perp Y \rightarrow$

$X \cdot Y = 0 \rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \rightarrow x_1 - x_2 = 0 \rightarrow$

$x_2 = t$ any # $\rightarrow$ $x_1 = t$, then

$Y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix}$ so $Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ works

(or any other value of t)

30.) $X = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, $Y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then $X \perp Y \rightarrow$

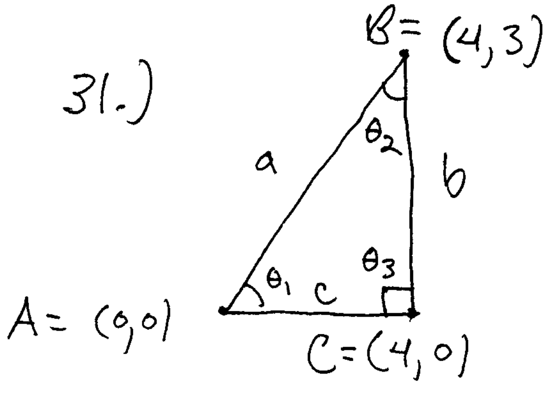
$X \cdot Y = 0 \rightarrow \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow 2x_1 - x_3 = 0 \rightarrow$

$x_3 = t$ any # $\rightarrow$ $2x_1 = t \rightarrow x_1 = \frac{1}{2}t$ and
$x_2 = s$ any #, then

$Y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}t \\ s \\ t \end{bmatrix}$, let $t = 2$, $s = 0$ (or any other values for t and s) $\rightarrow$

$Y = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ works

31.)



$a = \sqrt{(4-0)^2 + (3-0)^2} = 5$,
$b = 3$, $c = 4$; let

$\overrightarrow{AB} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$,

$\overrightarrow{BC} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$, then $\theta_3 = 90°$

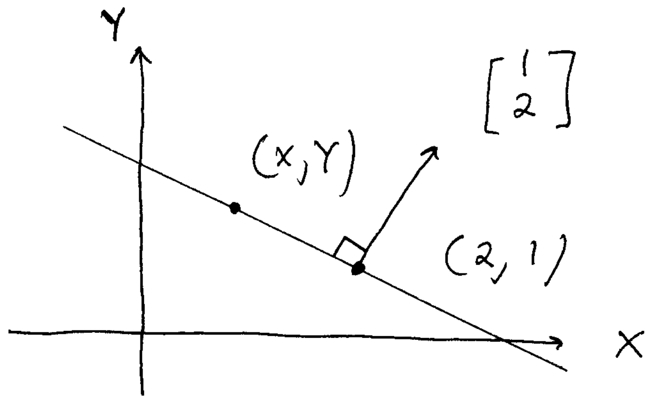
and $\cos\theta_1 = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|} = \dfrac{16+0}{(5)(4)} = \dfrac{4}{5} \rightarrow$

$\theta_1 = \arccos\left(\frac{4}{5}\right) \approx 0.644$ radians

or $36.9^\circ \rightarrow$

$\theta_2 = 180 - 90 - 36.9 = 53.1^\circ$

35.)



$\begin{bmatrix}1\\2\end{bmatrix}$ $\quad \begin{bmatrix}X-2\\Y-1\end{bmatrix} \perp \begin{bmatrix}1\\2\end{bmatrix} \rightarrow$

$\begin{bmatrix}X-2\\Y-1\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix} = 0 \rightarrow$

$(X-2)+2(Y-1)=0 \rightarrow X-2+2Y-2=0 \rightarrow$

$X+2Y=4$

39.) $(0)(X-1)+(-1)(Y-2)+(1)(Z-3)=0 \rightarrow$

$-Y+2+Z-3=0 \rightarrow Z-Y=1$

42.) $(-1)(X-3)+(1)(Y-(-1))+(2)(Z-2)=0 \rightarrow$

$-X+3+Y+1+2Z-4=0 \rightarrow$

$-X+Y+2Z=0$

43.) $L: \begin{cases} X = 1+2t \\ Y = -1+t \end{cases}$, $t =$ any #

47.) pts. $(-1,2)$ and $(3,4)$ $\rightarrow$ direction vector is $X = \begin{bmatrix}3-(-1)\\4-2\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$, then

$$L: \begin{cases} X = -1 + 4t \\ Y = 2 + 2t \end{cases} \quad ; \quad \text{then}$$

$$\left.\begin{matrix} X = -1 + 4t \\ Y = 2 + 2t \end{matrix}\right\} \rightarrow \left.\begin{matrix} X = -1 + 4t \\ 2Y = 4 + 4t \end{matrix}\right\} \rightarrow$$

$$2Y - X = 4 - (-1) + 4t - 4t \rightarrow$$

$$2Y - X = 5$$

53.) $2X + Y - 3 = 0$; let $X = t \rightarrow$

$2t + Y - 3 = 0 \rightarrow \quad Y = 3 - 2t \rightarrow$

$$L: \begin{cases} X = t \\ Y = 3 - 2t \end{cases}$$

55.) $$L: \begin{cases} X = 1 + t \\ Y = -1 + (-2)t \\ Z = 2 + t \end{cases}$$

58.) $$L: \begin{cases} X = 2 + 3t \\ Y = 1 - t \\ Z = -3 + 2t \end{cases}$$

59.) pts. $(5,4,-1)$, $(2,0,3) \rightarrow$ direction vector is $X = \begin{bmatrix} 5-2 \\ 4-0 \\ -1-3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$ so

$$L = \begin{cases} X = 5 + 3t \\ Y = 4 + 4t \\ Z = -1 - 4t \end{cases}$$

62.) pts. $(1,0,4), (3,2,0) \rightarrow$ direction vector is

$$X = \begin{bmatrix} 3-1 \\ 2-0 \\ 0-4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix} \text{ so } L: \begin{cases} x = 1 + 2t \\ y = 0 + 2t \\ z = 4 - 4t \end{cases}$$

63.) plane : $(1)(x-1) + (2)(y-(-1)) + (1)(z-2) = 0 \rightarrow$
$x - 1 + 2y + 2 + z - 2 = 0 \rightarrow$
$\boxed{x + 2y + z = 1}$; line : pts.

$(0,-3,2), (-1,-2,3) \rightarrow$ direction vector

$$X = \begin{bmatrix} -1-0 \\ -2-(-3) \\ 3-2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \text{ so } \boxed{L: \begin{cases} x = 0 - t \\ y = -3 + t \\ z = 2 + t \end{cases}} \; ;$$

intersection : $x + 2y + z = 1 \rightarrow$
$(-t) + 2(-3+t) + (2+t) = 1 \rightarrow$
$-t + {}^{-}6 + 2t + 2 + t = 1 \rightarrow$
$2t = 5 \rightarrow t = 5/2$ so plane and line intersect at pt.

$$\left(\frac{-5}{2}, -3 + \frac{5}{2}, 2 + \frac{5}{2}\right) = \left(\frac{-5}{2}, \frac{-1}{2}, \frac{9}{2}\right)$$

65.) plane : $(-1)(x-0) + (1)(y-(-2)) + (-1)(z-1) = 0 \rightarrow$
$-x + y + 2 - z + 1 = 0 \rightarrow \boxed{-x + y - z = {}^{-}3}$;
find 2 pts. in plane : $(0,-2,1)$ and $(3,0,0) \rightarrow$ direction vector is

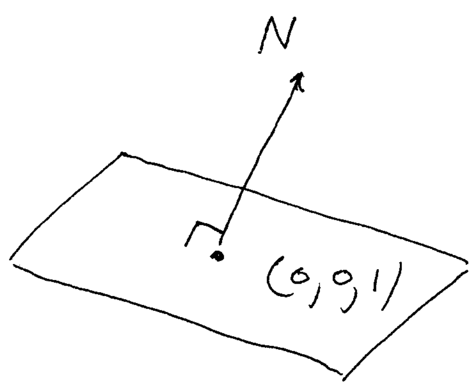
$$X = \begin{bmatrix} 3-0 \\ 0-(-2) \\ 0-1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$ and pt. $(5,-1,0)$ on line $\longrightarrow$

$$L: \begin{cases} x = 5 + 3t \\ y = -1 + 2t \\ z = 0 - t \end{cases}.$$

66.) plane $x + 2y - z + 1 = 0 \longrightarrow$

$(1)(x-0) + 2(y-0) + (-1)(z-1) = 0 \longrightarrow$

pt. on plane is $(0,0,1)$ and $\perp$ vector is $N = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$; so line through $(0,0,1)$ and parallel to N ($\perp$ to plane) is



$$L: \begin{cases} x = 0 + t \\ y = 0 + 2t \\ z = 1 - t \end{cases}$$