

Section 6.1

$$6.) \sum_{k=3}^5 (k-1)^2 = 2^2 + 3^2 + 4^2$$

$$7.) \sum_{k=2}^6 3^k = 3^2 + 3^3 + 3^4 + 3^5 + 3^6$$

$$8.) \sum_{k=1}^3 \frac{k^2}{k^2+1} = \frac{1}{2} + \frac{4}{5} + \frac{9}{10}$$

$$10.) \sum_{k=0}^4 k^x = 0^x + 1^x + 2^x + 3^x + 4^x$$

$$11.) \sum_{k=0}^3 (-1)^{k+1} = (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4$$

$$14.) \sum_{k=1}^n \cos\left(k \frac{\pi}{n}\right) \cdot \frac{\pi}{n}$$

$$= \cos\left(\frac{\pi}{n}\right) \cdot \frac{\pi}{n} + \cos\left(\frac{2\pi}{n}\right) \cdot \frac{\pi}{n} + \cos\left(\frac{3\pi}{n}\right) \cdot \frac{\pi}{n} \\ + \dots + \cos\left(\frac{n\pi}{n}\right) \cdot \frac{\pi}{n}$$

$$16.) \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} = \sum_{k=1}^4 \frac{1}{\sqrt{k}}$$

$$18.) \frac{3}{5} + \frac{4}{6} + \frac{5}{7} + \frac{6}{8} + \frac{7}{9} = \sum_{k=3}^7 \frac{k}{k+2}$$

$$22.) 1 - a + a^2 - a^3 + a^4 - a^5 + \dots + (-1)^n a^n$$

$$= \sum_{k=0}^n (-1)^k \cdot a^k$$

$$23.) \sum_{k=1}^{15} (2k+3) = 2 \left(\sum_{k=1}^{15} k \right) + \left(\sum_{k=1}^{15} 3 \right)$$

$$= 2 \cdot \frac{1}{2} (15)(16) + 3(15) = 285$$

$$25.) \sum_{k=0}^6 k(k+1) = \sum_{k=0}^6 (k^2+k) = \sum_{k=0}^6 k^2 + \sum_{k=0}^6 k$$

$$\begin{aligned}
 &= \sum_{k=1}^6 k^2 + \sum_{k=1}^6 k \\
 &= \frac{6(7)(13)}{6} + \frac{6(7)}{2} = 112
 \end{aligned}$$

$$\begin{aligned}
 27.) \quad \sum_{k=1}^n 4(k-1)^2 &= \sum_{k=1}^n 4(k^2 - 2k + 1) \\
 &= 4 \left(\sum_{k=1}^n k^2 \right) - 8 \left(\sum_{k=1}^n k \right) + \sum_{k=1}^n 4 \\
 &= 4 \cdot \frac{1}{6} n(n+1)(2n+1) - 8 \cdot \frac{1}{2} n(n+1) + 4n \\
 &= n \left[\frac{2}{3} (2n^2 + 3n + 1) - 4(n+1) + 4 \right] \\
 &= n \left[\frac{4}{3} n^2 + 2n + \frac{2}{3} - 4n - 4 + 4 \right] \\
 &= n \left[\frac{4}{3} n^2 - 2n + \frac{2}{3} \right] = \frac{2}{3} n [2n^2 - 3n + 1] \\
 &= \frac{2}{3} n (2n-1)(n-1)
 \end{aligned}$$

$$\begin{aligned}
 28.) \quad \sum_{k=1}^n (k+2)(k-2) &= \sum_{k=1}^n (k^2 - 4) = \sum_{k=1}^n k^2 - \sum_{k=1}^n 4 \\
 &= \frac{1}{6} n(n+1)(2n+1) - 4n \\
 &= \frac{1}{6} n [(2n^2 + 3n + 1) - 24] = \frac{1}{6} n (2n^2 + 3n - 23)
 \end{aligned}$$

$$\begin{aligned}
 30.) \quad \sum_{k=0}^{10} (-1)^k &= (-1)^0 + (-1)^1 + (-1)^2 + \dots + (-1)^{10} \\
 &= (1-1) + (1-1) + (1-1) + \dots + (1-1) + 1 = 1
 \end{aligned}$$