Section 6.2

5.) $y = \int_0^x \sqrt{1+2u}\, du \xrightarrow{D} \frac{dy}{dx} = \sqrt{1+2x}$

7.) $y = \int_0^x \sqrt{1+\sin^2 u}\, du \xrightarrow{D} \frac{dy}{dx} = \sqrt{1+\sin^2 x}$

12.) $y = \int_{-1}^x \frac{2}{2+u^2}\, du \xrightarrow{D} \frac{dy}{dx} = \frac{2}{2+x^2}$

15.) $y = \int_0^{3x} (1+t^2)\, dt \xrightarrow{D} \frac{dy}{dx} = (1+(3x)^2)\cdot(3)$

16.) $y = \int_0^{2x-1} (t^3-2)\, dt \xrightarrow{D} \frac{dy}{dx} = ((2x-1)^3-2)\cdot(2)$

20.) $y = \int_2^{x^2-2} \sqrt{3+u}\, du \xrightarrow{D} \frac{dy}{dx} = \sqrt{3+(x^2-2)}\cdot 2x$

24.) $y = \int_2^{\ln x} e^{-t}\, dt \xrightarrow{D} \frac{dy}{dx} = e^{-(\ln x)}\cdot \frac{1}{x}$

27.) $y = \int_{2x}^{3} (1+\sin t)\, dt = -\int_3^{2x} (1+\sin t)\, dt$

$\xrightarrow{D} \frac{dy}{dx} = -(1+\sin(2x))\cdot 2$

32.) $y = \int_{2+x^2}^{2} \cot t\, dt = -\int_2^{2+x^2} \cot t\, dt \xrightarrow{D}$

$\frac{dy}{dx} = -\cot(2+x^2)\cdot(2x)$

33.) $y = \int_x^{2x} (1+t^2)\, dt = \int_x^0 (1+t^2)\, dt + \int_0^{2x} (1+t^2)\, dt$

$= -\int_0^x (1+t^2)\, dt + \int_0^{2x} (1+t^2)\, dt \xrightarrow{D}$

$\frac{dy}{dx} = -(1+x^2) + (1+(2x)^2)\cdot 2$

36.) $y = \int_{x^3}^{x^4} \ln(1+t^2)\,dt$

$= \int_{x^3}^{0} \ln(1+t^2)\,dt + \int_{0}^{x^4} \ln(1+t^2)\,dt$

$= -\int_{0}^{x^3} \ln(1+t^2)\,dt + \int_{0}^{x^4} \ln(1+t^2)\,dt \xrightarrow{D}$

$\frac{dY}{dx} = -\ln(1+(x^3)^2)\cdot(3x^2) + \ln(1+(x^4)^2)\cdot(4x^3)$

40.) $\int (x^3 - 4)\,dx = \frac{1}{4}x^4 - 4x + C$

44.) $\int \left(\frac{1}{2}x^5 + 2x^3 - 1\right)dx = \frac{1}{2}\cdot\frac{1}{6}x^6 + 2\cdot\frac{1}{4}x^4 - x + C$

45.) $\int \frac{2x^2 - x}{\sqrt{x}}\,dx = \int \left[2\cdot\frac{x^2}{x^{1/2}} - \frac{x}{x^{1/2}}\right]dx$

$= \int \left[2\cdot x^{3/2} - x^{1/2}\right]dx = 2\cdot\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

50.) $\int (x^{3/5} + x^{5/3})\,dx = \frac{5}{8}x^{8/5} + \frac{3}{8}x^{8/3} + C$

51.) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int (x^{1/2} + x^{-1/2})\,dx = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

55.) $\int (x-2)(3-x)\,dx = \int (3x - x^2 - 6 + 2x)\,dx$

$= \int (-6 + 5x - x^2)\,dx = -6x + \frac{5}{2}x^2 - \frac{1}{3}x^3 + C$

57.) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$

62.) $\int e^x(1 - e^{-x})\,dx = \int (e^x - e^0)\,dx$

$= \int (e^x - 1)\,dx = e^x - x + C$

65.) $\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + C$

67.) $\int \sec^2(3x)\,dx = \frac{1}{3}\tan(3x) + c$

73.) $\int (\sec^2 x + \tan x)\,dx = \int \left(\sec^2 x + \frac{\sin x}{\cos x}\right) dx$

$= \int \left(\sec^2 x + \frac{-1}{\cos x} \cdot -\sin x\right) dx$

$= \tan x - \ln|\cos x| + c$

80.) $\int \frac{1}{x-3}\,dx = \ln|x-3| + c$

82.) $\int \frac{2x+5}{x}\,dx = \int \left[\frac{2x}{x} + \frac{5}{x}\right] dx$

$= \int \left[2 + 5 \cdot \frac{1}{x}\right] dx = 2x + 5\ln|x| + c$

88.) $\int \frac{2x^2}{1+x^2}\,dx = 2\int \frac{1+x^2-1}{1+x^2}\,dx$

$= 2\int \left[\frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}\right] dx = 2\int \left[1 - \frac{1}{1+x^2}\right] dx$

$= 2(x - \arctan x) + c$

89.) $\int 3^x\,dx = \frac{1}{\ln 3} \cdot 3^x + c$

92.) $\int 4^{-x}\,dx = \frac{-1}{\ln 4} \cdot 4^{-x} + c$

93.) $\int (x^2 + 2^x)\,dx = \frac{1}{3}x^3 + \frac{1}{\ln 2} \cdot 2^x + c$

95.) $\int (\sqrt{x} + \sqrt{e^x})\,dx = \int \left(x^{1/2} + (e^x)^{1/2}\right) dx$

$= \int \left(x^{1/2} + e^{\frac{1}{2}x}\right) dx = \frac{2}{3}x^{3/2} + 2e^{\frac{1}{2}x} + c$

97.) $\int_2^4 (3-2x)\,dx = (3x-x^2)\Big|_2^4$

$= (12-16)-(6-4) = -4-2 = -6$

99.) $\int_0^1 (x^3 - x^{1/3})\,dx$

105.) $\int_0^{\frac{\pi}{4}} \sin(2x)\,dx = \frac{-1}{2}\cos(2x)\Big|_0^{\frac{\pi}{4}}$

$= \frac{-1}{2}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \frac{-1}{2}\underbrace{\cos(0)}_{1} = 1/2$

108.) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{-1}{\cos x}\cdot -\sin x\,dx$

$= -\ln|\cos x|\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = -\ln\left|\cos\left(\frac{\pi}{4}\right)\right| - -\ln\left|\cos\left(-\frac{\pi}{4}\right)\right|$

$= -\ln\left(\frac{\sqrt{2}}{2}\right) + \ln\left(\frac{\sqrt{2}}{2}\right) = 0$

114.) $\int_{\frac{\pi}{20}}^{\frac{\pi}{15}} \sec(5x)\cdot\tan(5x)\,dx = \frac{1}{5}\sec(5x)\Big|_{\frac{\pi}{20}}^{\frac{\pi}{15}}$

$= \frac{1}{5}\sec\left(\frac{\pi}{3}\right) - \frac{1}{5}\sec\left(\frac{\pi}{4}\right)$

$= \frac{1}{5}\cdot\frac{2}{1} - \frac{1}{5}\cdot\frac{2}{\sqrt{2}} = \frac{2}{5} - \frac{\sqrt{2}}{5} = \frac{2-\sqrt{2}}{5}$

115.) $\int_{-1}^{0} e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_{-1}^{0} = \frac{1}{3}e^0 - \frac{1}{3}e^{-3}$

$= \frac{1}{3} - \frac{1}{3}e^{-3}$

116.) $\int_0^2 2te^{t^2}\,dt = e^{t^2}\Big|_0^2 = e^4 - 1$

119.) $\int_1^e \frac{1}{x}\,dx = \ln|x|\Big|_1^e = \cancel{\ln} e - \cancel{\ln} 1 = 1$

123.) $\lim\limits_{x\to 0} \dfrac{\int_0^x \sin t\,dt}{x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{D \int_0^x \sin t\,dt}{2x}$

$= \lim\limits_{x\to 0} \dfrac{\sin x}{2x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \dfrac{\cos 0}{2} = \dfrac{1}{2}$

126.) $\int_0^x f(t)\,dt = \frac{1}{2}\tan(2x) \xrightarrow{D}$

$f(x) = \frac{1}{\cancel{2}} \cdot \sec^2(2x) \cdot \cancel{2} \longrightarrow$

$f(x) = \sec^2(2x)$