Section 7.1

1.) $\int 2x\sqrt{x^2+3}\,dx$ (Let $u = x^2+3 \xrightarrow{D}$ $du = 2x\,dx$)

$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + c = \frac{2}{3}(x^2+3)^{3/2} + c$

4.) $\int 4x^3(4+x^4)^{1/3}dx$ (Let $u = 4+x^4 \xrightarrow{D} du = 4x^3dx$)

$= \int u^{1/3}du = \frac{3}{4}u^{4/3} + c = \frac{3}{4}(4+x^4)^{4/3} + c$

6.) $\int 5\sin(1-2x)\,dx$ (Let $u = 1-2x \xrightarrow{D}$ $du = -2\,dx \rightarrow \frac{-1}{2}du = dx$)

$= 5 \cdot \frac{-1}{2}\int \sin u\,du = \frac{-5}{2} \cdot -\cos u + c$

$= \frac{5}{2}\cos(1-2x) + c$

8.) $\int x\cos(x^2-1)\,dx$ (Let $u = x^2-1 \xrightarrow{D}$ $du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + c = \frac{1}{2}\sin(x^2-1) + c$

9.) $\int e^{2x+3}dx$ (Let $u = 2x+3 \xrightarrow{D}$ $du = 2\,dx \rightarrow \frac{1}{2}du = dx$)

$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{2x+3} + c$

12.) $\int xe^{1-3x^2}dx$ (Let $u = 1-3x^2 \xrightarrow{D}$ $du = -6x\,dx \rightarrow \frac{-1}{6}du = x\,dx$)

$$= \frac{-1}{6}\int e^u\,du = \frac{-1}{6}e^u + c = \frac{-1}{6}e^{1-3x^2} + c$$

13.) $\int \frac{x+2}{x^2+4x}\,dx$ (Let $u = x^2+4x \xrightarrow{D}$
$du = (2x+4)\,dx = 2(x+2)\,dx \rightarrow \frac{1}{2}du = (x+2)\,dx$)

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|x^2+4x| + c$$

14.) $\int \frac{2x}{3-x^2}\,dx$ (Let $u = 3-x^2 \xrightarrow{D}$
$du = -2x\,dx \rightarrow -du = 2x\,dx$)

$$= -\int \frac{1}{u}\,du = -\ln|u| + c = -\ln|3-x^2| + c$$

16.) $\int \frac{x}{5-x}\,dx$ (Let $u = 5-x \xrightarrow{D} du = -dx \rightarrow$
$-du = dx$ and $x = 5-u$)

$$= -\int \frac{5-u}{u}\,du = -\int \left[\frac{5}{u} - 1\right] du = -(5\ln|u| - u) + c$$

$$= -5\ln|5-x| + (5-x) + c$$

18.) $\int (4-x)^{1/7}\,dx$ (Let $u = 4-x \rightarrow du = -dx$
$\rightarrow -du = dx$)

$$= -\int u^{1/7}\,du$$

$$= -\frac{7}{8}u^{8/7} + c = -\frac{7}{8}(4-x)^{8/7} + c$$

20.) $\int (x^2-2x)(x^3-3x^2+3)^{2/3}\,dx$
(Let $u = x^3-3x^2+3 \xrightarrow{D} du = (3x^2-6x)\,dx \rightarrow$
$du = 3(x^2-2x)\,dx \rightarrow \frac{1}{3}du = (x^2-2x)\,dx$)

$$= \frac{1}{3}\int u^{2/3}\,du = \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{5}u^{5/3} + c$$

$$= \frac{1}{5}(x^3-3x^2+3)^{5/3} + c$$

21.) $\int \frac{x-1}{1+4x-2x^2}\,dx$ (Let $u = 1+4x-2x^2 \xrightarrow{D}$
$du = (4-4x)\,dx = -4(x-1)\,dx \rightarrow$
$-\frac{1}{4}\,du = (x-1)\,dx$)
$= -\frac{1}{4}\int \frac{1}{u}\,du = \frac{-1}{4}\ln|u| + c = \frac{-1}{4}\ln|1+4x-2x^2| + c$

24.) $\int \frac{x^3-1}{x^4-4x}\,dx$ (Let $u = x^4-4x \xrightarrow{D}$
$du = (4x^3-4)\,dx = 4(x^3-1)\,dx \rightarrow$
$\frac{1}{4}\,du = (x^3-1)\,dx$)
$= \frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}\cdot\ln|u| + c = \frac{1}{4}\ln|x^4-4x| + c$

26.) $\int \cos x \cdot e^{\sin x}\,dx$ (Let $u = \sin x \xrightarrow{D}$
$du = \cos x\,dx$)
$= \int e^u\,du = e^u + c = e^{\sin x} + c$

27.) $\int \frac{1}{x}\csc^2(\ln x)\,dx$ (Let $u = \ln x \xrightarrow{D} du = \frac{1}{x}\,dx$)
$= \int \csc^2 u \cdot du = -\cot u + c = -\cot(\ln x) + c$

28.) $\int \sec^2 x \cdot e^{\tan x}\,dx$ (Let $u = \tan x \xrightarrow{D}$
$du = \sec^2 x\,dx$)
$= \int e^u\,du = e^u + c = e^{\tan x} + c$

31.) $\int \tan x \cdot \sec^2 x\,dx$ (Let $u = \tan x \xrightarrow{D}$
$du = \sec^2 x\,dx$)
$= \int u\,du = \frac{1}{2}u^2 + c = \frac{1}{2}(\tan x)^2 + c$

32.) $\int \sin^3 x \cdot \cos x \, dx$ (Let $u = \sin x \xrightarrow{D} du = \cos x \, dx$)

$= \int u^3 \, du = \frac{1}{4} u^4 + c = \frac{1}{4} (\sin x)^4 + c$

33.) $\int \frac{(\ln x)^2}{x} dx$ (Let $u = \ln x \xrightarrow{D}$

$du = \frac{1}{x} dx$

$= \int u^2 du = \frac{1}{3} u^3 + c = \frac{1}{3} (\ln x)^3 + c$

35.) $\int x^3 \sqrt{5 + x^2} \, dx = \int x \cdot x^2 \sqrt{5 + x^2} \, dx$

(Let $u = 5 + x^2 \xrightarrow{D} du = 2x \, dx \rightarrow \frac{1}{2} du = x \, dx$ and $x^2 = u - 5$)

$= \frac{1}{2} \int (u - 5) u^{1/2} du = \frac{1}{2} \int (u^{3/2} - 5u^{1/2}) \, du$

$= \frac{1}{2} \left(\frac{2}{5} u^{5/2} - 5 \cdot \frac{2}{3} u^{3/2} \right) + c$

$= \frac{1}{5} (5 + x^2)^{5/2} - \frac{5}{3} (5 + x^2)^{3/2} + c$

36.) $\int \sqrt{1 + \ln x} \cdot \frac{\ln x}{x} dx$ (Let $u = 1 + \ln x \xrightarrow{D}$

$du = \frac{1}{x} dx$ and $\ln x = u - 1$)

$= \int \sqrt{u} \cdot (u - 1) \, du = \int (u^{3/2} - u^{1/2}) \, du$

$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + c$

$= \frac{2}{5} (1 + \ln x)^{5/2} - \frac{2}{3} (1 + \ln x)^{3/2} + c$

43.) $\int_0^3 x\sqrt{x^2+1}\,dx$ (Let $u = x^2+1 \xrightarrow{D}$

$du = 2x\,dx$; $x: 0 \to 3$ so $u: 1 \to 10$

$\frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int_1^{10}\sqrt{u}\,du = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_1^{10} = \frac{1}{3}\cdot 10^{3/2} - \frac{1}{3}\cdot 1^{3/2}$

$= \frac{1}{3}\cdot 10^{3/2} - \frac{1}{3}$

45.) $\int_2^3 \frac{2x+3}{(x^2+3x)^3}\,dx$ (Let $u = x^2+3x \xrightarrow{D}$

$du = (2x+3)\,dx$; $x: 2 \to 3$ so $u: 10 \to 18$)

$= \int_{10}^{18} \frac{1}{u^3}\,du = \int_{10}^{18} u^{-3}\,du = \frac{-1}{2}u^{-2}\Big|_{10}^{18}$

$= \frac{-1}{2}\cdot\frac{1}{18^2} - \frac{-1}{2}\cdot\frac{1}{10^2} = \frac{-1}{648} + \frac{1}{200}$

$= \frac{-25}{16{,}200} + \frac{81}{16{,}200} = \frac{56}{16{,}200}$

48.) $\int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2}\,dx$ (Let $u = e^x - 3 \xrightarrow{D}$

$du = e^x\,dx$; $x: \ln 4 \to \ln 7$ so

$u: (e^{\ln 4} - 3) \to (e^{\ln 7} - 3)$, i.e., $u: 1 \to 4$)

$= \int_1^4 \frac{1}{u^2}\,du = \frac{-1}{u}\Big|_1^4 = \frac{-1}{4} - -1 = \frac{3}{4}$

50.) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^2 x \cos x\,dx$ (Let $u = \sin x \xrightarrow{D}$

$du = \cos x\,dx$; $x: -\frac{\pi}{6} \to \frac{\pi}{6}$ so

$u: \sin\left(-\frac{\pi}{6}\right) \to \sin\frac{\pi}{6}$, i.e.,
$u = \frac{-1}{2} \to \frac{1}{2}$)

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} u^2\,du = \frac{1}{3}u^3 \Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{3}\left(-\frac{1}{2}\right)^3$$

$$= \frac{1}{24} + \frac{1}{24} = \frac{2}{24} = \frac{1}{12}$$

52.) $\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x}\,dx$ (Let $u = \cos x \xrightarrow{D}$

$du = -\sin x\,dx \to -du = \sin x\,dx$;
$x: 0 \to \frac{\pi}{3}$ so $u: \cos 0 \to \cos\frac{\pi}{3}$, i.e.,
$u: 1 \to 1/2$)

$$= -\int_1^{1/2} \frac{1}{u^2}\,du = -\left(\frac{-1}{u}\right)\Big|_1^{1/2} = \frac{1}{u}\Big|_1^{1/2}$$

$$= 2 - 1 = 1$$

54.) $\int_0^2 \frac{x}{x+2}\,dx$ (Let $u = x+2 \xrightarrow{D} du = 1\,dx$
and $x = u-2$; $x: 0 \to 2$ so $u: 2 \to 4$)

$$= \int_2^4 \frac{u-2}{u}\,du = \int_2^4 \left(1 - \frac{2}{u}\right) du$$

$$= (u - 2\ln|u|)\Big|_2^4 = (4 - 2\ln 4) - (2 - 2\ln 2)$$

$$= 4 - 2\ln 4 - 2 + \ln 2^2$$

$$= 2 - 2\ln 4 + \ln 4 = 2 - \ln 4$$

55.) $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx$ (Let $u = \ln x \xrightarrow{D}$

$du = \frac{1}{x}\,dx$; $x: e \to e^2$ so

$u: \ln e \to \ln e^2$, i.e., $u: 1 \to 2$)

$$= \int_1^2 \frac{1}{u^2}\,du = \frac{-1}{u}\Big|_1^2 = \frac{-1}{2} - -1 = \frac{1}{2}$$

57.) $\displaystyle\int_1^9 \frac{1}{\sqrt{x}} e^{-\sqrt{x}}\,dx$ (Let $u = -\sqrt{x} \xrightarrow{D}$

$du = -\frac{1}{2}x^{-1/2}dx \to -2\,du = \frac{1}{\sqrt{x}}\,dx$;

$x: 1 \to 9$ so $u: -1 \to -3$)

$$= -2\int_{-1}^{-3} e^u\,du = -2e^u\Big|_{-1}^{-3}$$

$$= -2e^{-3} - -2e^{-1} = 2e^{-1} - 2e^{-3}$$