Section 7.2

1.) $\int x \cos x \, dx$ (Let $u = x$, $dV = \cos x \, dx$
$\rightarrow du = 1 \, dx$, $V = \sin x$)

$= x \sin x - \int \sin x \, dx$

$= x \sin x - (-\cos x) + C$

$= x \sin x + \cos x + C$

6.) $\int x \sin(1-2x) dx$ (Let $u = x$, $dv = \sin(1-2x) dx$
$\rightarrow du = 1 dx$, $V = \frac{1}{2} \cos(1-2x)$)

$= \frac{1}{2} x \cos(1-2x) - \frac{1}{2} \int \cos(1-2x) \, dx$

$= \frac{1}{2} x \cos(1-2x) - \frac{1}{2} \cdot \frac{-1}{2} \sin(1-2x) + C$

7.) $\int x e^x dx$ (Let $u = x$, $dV = e^x dx$
$\rightarrow du = 1 \, dx$, $V = e^x$)

$= xe^x - \int e^x dx = xe^x - e^x + C$

10.) $\int 2x^2 e^{-x} dx$ (Let $u = 2x^2$, $dV = e^{-x} dx$
$\rightarrow du = 4x \, dx$, $V = -e^{-x}$)

$= -2x^2 e^{-x} - -4 \int x e^{-x} dx$

(Let $u = x$, $dV = e^{-x} dx \rightarrow$
$du = 1 \, dx$, $V = -e^{-x}$)

$= -2x^2 e^{-x} + 4 \left[ -xe^{-x} - - \int e^{-x} dx \right]$

$= -2x^2e^{-x} - 4xe^{-x} + 4(-e^{-x}) + c$

$= -2x^2e^{-x} - 4xe^{-x} - 4e^{-x} + c$

12.) $\int x^2 \ln x \, dx$ (Let $u = \ln x$, $dv = x^2 dx$
$\rightarrow du = \frac{1}{x} dx$, $v = \frac{1}{3}x^3$)

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3}x^3 + c$

15.) $\int x \sec^2 x \, dx$ (Let $u = x$, $dv = \sec^2 x \, dx$
$\rightarrow du = 1 dx$, $v = \tan x$)

$= x \tan x - \int \tan x \, dx$

$= x \tan x - \ln|\sec x| + c$

19.) $\int_1^2 \ln x \, dx$ (Let $u = \ln x$, $dv = dx \rightarrow$
$du = \frac{1}{x} dx$, $v = x$)

$= x \ln x \Big|_1^2 - \int_1^2 1 \, dx$

$= 2\ln 2 - 1 \cdot \cancel{\ln 1}^{0} - x\Big|_1^2$

$= \ln 2^2 - (2-1) = \ln 4 - 1$

23.) $\int_0^1 xe^{-x} dx$ (Let $u = x$, $dv = e^{-x} dx \rightarrow$
$du = 1 dx$, $v = -e^{-x}$)

$= -xe^{-x}\Big|_0^1 - -\int_0^1 e^{-x} dx$

$= (-e^{-1} - 0) + (-e^{-x})\Big|_0^1 = \frac{-1}{e} + \left(\frac{-1}{e} - -\cancel{e^0}^{1}\right)$

$= 1 - 2/e$

25.) $\underline{\underline{\int e^x \sin x \, dx}}$ (Let $u = e^x$, $dv = \sin x \, dx$
$\rightarrow du = e^x dx$, $v = -\cos x$ )

$= -e^x \cos x - -\int e^x \cos x \, dx$

$= -e^x \cos x + \int e^x \cos x \, dx$

(Let $u = e^x$, $dv = \cos x \, dx$
$\rightarrow du = e^x dx$, $v = \sin x$ )

$= -e^x \cos x + e^x \sin x - \underline{\underline{\int e^x \sin x \, dx}}$ ; then

$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x + C \rightarrow$

$\int e^x \sin x \, dx = \frac{-1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C \rightarrow$

$\int_0^{\frac{\pi}{3}} e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \Big|_0^{\frac{\pi}{3}}$

$= \frac{1}{2} e^{\pi/3} (\sin \frac{\pi}{3} - \cos \frac{\pi}{3}) - \frac{1}{2} e^0 (\sin 0 - \cos 0)$

$= \frac{1}{2} e^{\pi/3} \left( \frac{\sqrt{3}}{2} - \frac{1}{2} \right) - \frac{1}{2}(0 - 1)$

$= \frac{\sqrt{3} - 1}{4} \cdot e^{\pi/3} + \frac{1}{2}$

29.) $\underline{\underline{\int \sin(\ln x) \, dx}}$ (Let $u = \sin(\ln x)$, $dv = dx$.
$\rightarrow du = \cos(\ln x) \cdot \frac{1}{x} dx$, $v = x$)

$= x \sin(\ln x) - \int \cos(\ln x) \, dx$

(Let $u = \cos(\ln x)$, $dv = dx$ $\rightarrow$
$du = -\sin(\ln x) \cdot \frac{1}{x} dx$, $v = x$)

$= x\sin(\ln x) - [x\cos(\ln x) - {}^{-}\int \sin(\ln x)\,dx]$

$= x\sin(\ln x) - x\cos(\ln x) - \underline{\underline{\int \sin(\ln x)\,dx}}$ ;

then $2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + C$

$\rightarrow \int \sin(\ln x)\,dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$

40.) $\int \sin\sqrt{x}\,dx$ (Let $x = u^2 \xrightarrow{D} dx = 2u\,du$)
$\hookrightarrow u = \sqrt{x}$

$= 2\int u\sin u\,du$ (Let $w = u$, $dv = \sin u\,du$
$\rightarrow dw = 1\,du$, $v = -\cos u$)

$= 2[-u\cos u - {}^{-}\int \cos u\,du]$

$= -2u\cos u + 2\sin u + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

43.) $\int \sin x\cos x\,e^{\sin x}\,dx$
(Let $u = \sin x \xrightarrow{D} du = \cos x\,dx$)

$= \int u e^u\,du$ (Let $w = u$, $dv = e^u\,du$
$\rightarrow dw = 1\,du$, $v = e^u$)

$= ue^u - \int e^u\,du = ue^u - e^u + C$

$= \sin x\,e^{\sin x} - e^{\sin x} + C$

45.) $\int_0^1 e^{\sqrt{x}}\,dx$ (Let $x = u^2 \rightarrow u = \sqrt{x}$ and $dx = 2u\,du$)

$= 2\int_{x=0}^{x=1} u e^u\,du$ (Let $w = u$, $dv = e^u\,du$ $\longrightarrow$ $dw = 1\,du$, $v = e^u$)

$= 2\left[u e^u\Big|_{x=0}^{x=1} - \int_0^1 e^u\,du\right]$

$= \left(2ue^u - 2\cdot e^u\right)\Big|_{x=0}^{x=1}$

$= \left(2\sqrt{x}\,e^{\sqrt{x}} - 2\cdot e^{\sqrt{x}}\right)\Big|_0^1 = (2e-2e)-(0-2e^0)$

$= 2$

48.) $\int_0^1 x^3 \ln(x^2+1)\,dx$ (Let $u = \ln(x^2+1)$, $dv = x^3\,dx$ $\rightarrow du = \frac{2x}{x^2+1}\,dx$, $v = \frac{1}{4}x^4$)

$= \frac{1}{4}x^4\ln(x^2+1)\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{x^5}{x^2+1}\,dx$

$= \frac{1}{4}\ln 2 - 0$

$-\frac{1}{2}\int_0^1 \left[x^3 - x + \frac{x}{x^2+1}\right]dx$

$$\begin{array}{r|l} & x^3 - x \\ \hline x^2+1 & x^5 \\ & -(x^5+x^3) \\ \hline & -x^3 \\ & -(-x^3-x) \\ \hline & x \end{array}$$

$= \frac{1}{4}\ln 2$

$-\frac{1}{2}\left(\frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{1}{2}\ln(x^2+1)\right)\Big|_0^1$

$= \frac{1}{4}\ln 2 - \frac{1}{2}\left[\left(\frac{1}{4} - \frac{1}{2} + \frac{1}{2}\ln 2\right) - \left(0 - 0 + \frac{1}{2}\cancel{\ln 1}^{\,0}\right)\right]$

$= \cancel{\frac{1}{4}\ln 2} - \frac{1}{8} + \frac{1}{4} - \cancel{\frac{1}{4}\ln 2} = \frac{1}{8}$

49.) $\int xe^{-2x}\,dx$ (Let $u = x$, $dv = e^{-2x}\,dx$
$\rightarrow du = 1\,dx$, $v = \frac{-1}{2}e^{-2x}$)

$= \frac{-1}{2}xe^{-2x} - \frac{-1}{2}\int e^{-2x}\,dx$

$= \frac{-1}{2}xe^{-2x} + \frac{1}{2}\cdot\frac{-1}{2}e^{-2x} + c$

50.) $\int xe^{-2x^2}\,dx$ (Let $u = -2x^2 \xrightarrow{D}$
$du = -4x\,dx \rightarrow \frac{-1}{4}du = x\,dx$)

$= \frac{-1}{4}\int e^u\,du = \frac{-1}{4}e^u + c = \frac{-1}{4}e^{-2x^2} + c$

51.) $\int \frac{1}{\tan x}\,dx = \int \cot x\,dx = \ln|\sin x| + c$

52.) $\int \frac{1}{\csc x \cdot \sec x}\,dx = \int \frac{1}{\frac{1}{\sin x}\cdot\frac{1}{\cos x}}\,dx$

$= \int \sin x \cos x\,dx$ (Let $u = \sin x \xrightarrow{D}$
$du = \cos x\,dx$)

$= \int u\,du = \frac{1}{2}u^2 + c = \frac{1}{2}(\sin x)^2 + c$

53.) $\int 2x\sin(x^2)\,dx$ (Let $u = x^2 \xrightarrow{D}$
$du = 2x\,dx$)

$= \int \sin u\,du = -\cos u + c = -\cos(x^2) + c$

54.) $\int 2x^2 \sin x\,dx$ (Let $u = 2x^2$, $dv = \sin x\,dx$
$\rightarrow du = 4x\,dx$, $v = -\cos x$)

$= -2x^2\cos x - {}^-4\int x\cos x\,dx$

(Let $u = x$, $dv = \cos x\,dx$
$\rightarrow du = 1\,dx$, $v = \sin x$)

$= -2x^2 \cos x + 4\left[x \sin x - \int \sin x \, dx\right]$

$= -2x^2 \cos x + 4x \sin x - 4\cdot(-\cos x) + C$

55.) $\int \frac{1}{4^2+x^2}\,dx = \frac{1}{4} \arctan\left(\frac{x}{4}\right) + C$

56.) $\int \frac{1}{x^2+(\sqrt{5})^2}\,dx = \frac{1}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right) + C$

57.) $\int \frac{x}{x+3}\,dx$ (Let $u = x+3 \xrightarrow{D} du = 1\,dx$
and $x = u-3$)

$= \int \frac{u-3}{u}\,du = \int \left(1 - \frac{3}{u}\right) du$

$= u - 3\ln|u| + C = x+3 - 3\ln|x+3| + C$

59.) $\int \frac{x}{x^2+3}\,dx$ (Let $u = x^2+3 \xrightarrow{D}$
$du = 2x\,dx \longrightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+3| + C$

60.) $\int \frac{x+2}{x^2+2}\,dx = \int \frac{x}{x^2+2}\,dx + \int \frac{2}{x^2+2}\,dx$

(Let $u = x^2+2 \xrightarrow{D} du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$= \frac{1}{2}\int \frac{1}{u}\,du + 2\int \frac{1}{x^2+(\sqrt{2})^2}\,dx$

$= \frac{1}{2}\ln|u| + 2\cdot\frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$

$= \frac{1}{2}\ln|x^2+2| + \sqrt{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$

63.) $\int x(x-2)^{1/4}\,dx$ (Let $u = x-2 \xrightarrow{D}$
$du = 1\,dx$ and $x = u+2$)

$= \int (u+2)\,u^{1/4}\,du = \int (u^{5/4} + 2\cdot u^{1/4})\,du$

$= \frac{4}{9}u^{9/4} + 2\cdot\frac{4}{5}u^{5/4} + C = \frac{4}{9}(x-2)^{9/4} + \frac{8}{5}(x-2)^{5/4} + C$

68.) $\int_1^2 x^2 \ln x\,dx$ (Let $u = \ln x$, $dv = x^2\,dx$
$\rightarrow du = \frac{1}{x}\,dx$, $v = \frac{1}{3}x^3$)

$= \frac{1}{3}x^3 \ln x \Big|_1^2 - \frac{1}{3}\int_1^2 x^2\,dx$

$= \frac{1}{3}(8)\ln 2 - \frac{1}{3}(1)\cancel{\ln 1}^{0} - \frac{1}{3}\cdot\frac{1}{3}x^3\Big|_1^2$

$= \frac{8}{3}\ln 2 - \frac{1}{9}(8-1) = \frac{8}{3}\ln 2 - \frac{7}{9}$